Least Possible Time to reach Targeted Profit Function under Unitary Transformation

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Abstract: *The problem of quickest descent is solved by the calculus of variations. The calculus of variations is a branch of mathematics dealing with the optimization problem of physical quantities. Here profit maximization problems are judged by using this idea. As it is known that profit velocity and the time are the key factors to optimize the policy so, we have investigated the path of profit function and the minimum time to reach the final destination of profit function by utilizing unitary operator [1].*

Given two states that is the starting profit function and the targeted profit function there exist different paths belonging to the set. This investigation uses the unitary transformation which transforms the starting profit function to the targeted profit function in the least possible time.

Keywords: *Profit Velocity; Targeted Profit Function; Unitary Operator.*

JEL Classifications: *C 61; D 21.*

Introduction

The application of unitary operators into economics was brought by the study of the targeted profit function that arise in any mathematical theory of an economic system and is closely related with profit velocity that is profit generated per minute of production and optimal timing. The purpose of this paper is to present a detailed discussion of different paths of targeted profit functions and then to make a related approaches in this arena.

How quickly can we reach the maximum targeted profit function and by which transformation? We say that a problem is in time function if there is a unitary

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transformation which decides whether a profit function is in the language in time and makes its maximum profit. It laid the analytical foundations for the study of allocation of time within the household (Becker, G. S 1965), corporate and for national economy (Caragea, N 2009). L. A. Boland (Boland, L. A 1978) have established the fact that neoclassical economics is not timeless but it treats time explicitly. A necessary and sufficient condition for time consistently undominated policies under Ramsey policies which are equilivalent to first order conditions has been derived by Brendon & Ellison (Brendon, C & Ellison, M 2018).

Huang, Leng and Parlar (Huang et al, 2013) provide a comprehensive survey of commonly used demand models which depend on time, space, quality etc. It is applied on price sensitivity and the taste of differentiation in the monopoly and duopoly setting (Chambers et al, 2006 and Desai, P 2001). Its continuing relevance in institutional, territorial (Corpataux, J & Crevoisier, O 2007) and empirical economics is a testimony of power (Heckman, J 2015). The basic issue concerning to attain targeted profit in a competitive world is whether a market solution will yield the socially optimum kinds and quantities of commodities.

Let π be a complete mathematical description of the profit function to be studied (i.e., π might be comprised of difference between revenue function and cost function). Assumptions made a prior about π (assumption of continuity of revenue function and cost function), define the space Π of profit functions to which the study is restricted. By a revenue function or a cost function we mean a specific values of all the relevant endogenous variables (i.e., for revenue function we mean income of consumers, quality of the good, price etc., and by cost function it means cost price, wealth, government policies, tax rate, labor, capital etc.).

As a first test of sufficiency of this economic model, it must be possible to prove that for every element π of an adequately extensive class Π, the set Π is not empty and that means it obeys the existence problem. The mathematical tools for the solution were provided by unitary transformations in the form of linear operator (we have used matrix operators) or directly related results. The mathematical techniques applied here were those of functional analysis.

Having obtained a general solution to the targeted profit function one must investigate the structure of Π (π_T) of targeted profit function π_T. If we consider an economy that has a targeted profit function near π_T such that in any neighborhood of π_T, there are infinitely many other profit functions related with different type of transformations. In this situation the targeted profit function is indeterminate near π_T. Moreover, the economic system π_T is unstable in this sense that arbitrary small perturbation from π_T to a neighboring profit functions induce no tendency for the state of the economy to return to π_T. It is therefore desirable to have a function around π_T is discrete; such that for every π_T in Π there is a neighborhood of π_T is the unique solution. But if all the conditions here prevail mathematically well behaved one may obtain a set Π that may not be discrete.

There is another type of problem to determine targeted profit function by using transformation. Actually there are different types of transformations which transform a profit function to different targeted profit functions. Specifically if π is close to π' then the application of transformations T_1 and T_2 on π and π' respectively leads to T_1(π) and T_2(π') and they are close together. This error to get desired targeted profit function would yield entirely different values and thus destroy the explanatory power of the theory also. Therefore, it is also desirable to choose a special type of transformation to be such that the application of transformation on starting profit function reaches to the desired targeted profit function only without disabling the system.

This difficulty is solved by functional analysis by using unitary transformations. As it is known that there are many paths to join the two points in the Real Space, there exist many transformations to obtain the desired result. But our aim is to obtain the targeted profit without hampering the dynamical system in the space Π, so we have chosen unitary transformation as it is known that a unitary transformation transforms a linear operator into a linear operator and leaves invariant any algebraic equation between linear operators.

In the first model, we observe that the least possible time is dependent on profit velocity and cost function. This case indicates that the targeted profit function is achieved by studying only the costs function and profit velocity, ignoring the revenue function and cost increasing parameter. In the second model, the time is dependent on the profit velocity and the revenue function. We have put a parameter here, the revenue increasing parameter, but it has no effect in the least possible time. In the case of model three, the least possible time is dependent on profit velocity, revenue increasing parameter and cost function by surpassing the cost increasing parameter.

Our paper explores the relation between the profit velocity and the time of the shortest path between the starting profit function and the targeted profit function by using unitary transformation. Unitary transformation gives impulses on the starting profit function of a firm to get the final profit function in a least possible time.

Model:

As it is known that profit of a firm is dependent on demand and cost function, so it can be expressed in the following form:

$$\pi = R - C,$$

where π denotes profit of a firm, R is a revenue function and C is a cost function.

Supposing a new firm has tried to enter in a competition with the potential big bosses for a new innovation which will reduce costs and give maximum profit in a short span of time. The investor will try to understand the dynamics of the economy by observing the conditions of more investment in the research and development area, velocity of profit function and time factor to attain the maximum profit in a short span of time without giving chances to his opponents to understand his economic policy. But the problem arises here if the investor will engage in direct completion with his opponents then the rent dissipation effect and replacement effects come in action and this problem is solved by Kenneth Arrow in 1962 (Arrow, K 1962) (which is totally against the new investor) and is well discussed by Aghion and Griffith (Aghion & Griffith 2005) (here the new investor may win the race by investing more than the old big bosses). But our investor will not be interested in this type of direct conflict and he will try to win the race by giving importance in time factor. Now the problem is how does this relate to attain the maximum profit in the quickest time? So, let us plunge our attention to our model where the problems are discussed elaborately.

Here, one empirical researcher may face some of the methodological difficulties because those were not dealt with due to lack of data in this area.

First, it resulted to be important to control own firm and to investigate characteristics of the least possible time that affect profit function. The reasons behind this are the characteristics of the cost-dependent parameter of the firm and the dynamics of profit velocity. For example, as we know that least possible time to reach targeted profit function is related with cost-dependent parameter and profit velocity and this parameter is positively related with cost function, but we cannot reach targeted profit of the firm by enforcing the parameter only, then it could be the case that least possible time to reach targeted profit is genuinely related with profit velocity and cost function only and here the cost-dependent parameter plays no role. But we have to keep in mind that unless we manage for at least the main observables and unobservable characteristics, we cannot be sure to predict the relationship among time with profit velocity and cost function with the application of unitary transformation.

Second, there is a problem of different causality. While profit velocity and cost function are likely to affect the least possible time for the targeted function, it is also the case that revenue function also affects the least possible time. Firms that are engaged in innovation always try to reduce costs (to be able to sell at a lower price) or have to increase revenue (by producing superior quality goods and others) and in either case will command market share. But when firms are unable to reduce the total cost significantly, then firms rely on revenue function (i.e. on price, income etc) to mobilize profit function. What is important is that there is variation in the profit function; for example, policy changes (here we take unitary operators as a symbol of policy change) that make firms attain targeted profit in least possible time.

Third, the relationship we are interested in is among time, profit velocity, revenue-increasing factor and cost function where early investigations are mainly concentrated largely on price sensitivity, quality and demand functions etc. These may create problems because the competition may affect price cost concentration. It can be difficult to obtain time factor to attain targeted profit function in an industry. Our work is mainly concentrated on the transformation of a system without hampering the other factors in a firm particularly geographic and product areas in which the firm operates. And here unitary operators play an important role because this is particularly important in applications where firms operate in big markets, so that traditional ways of application to study the market behavior could be misleading.

Let us assume that π_S be a starting profit and π_T be targeted profit of a firm and we have to recall that π_T is reached from the initial profit π_S through the shortest path at time 't'. We will find out the time 't' and the related conditions to attain the target point. It is possible to construct different paths to join π_S and π_T but we have assumed the shortest path between the two points.

We have taken two postulates for our models to set up the space in which our economy takes place. This arena is from linear algebra.

Postulate 1: Associated to any isolated economical system in our model is a real valued inner product space that is a complete metric space with respect to distance function induced by inner product space.

Postulate 2: The evolution of our economic system is described by a unitary transformation. That is the position of starting profit function π_S at time t_1 is related to the state π_T at time t_2 by a unitary operator U_t which depends only on the times t_1 and t_2, and t = t_2 - t_1.

We know from postulate 2 that the targeted profit is produced by motion of the dynamics of the firm policy acting on the starting profit and the operator acting on the starting profit is a unitary operator (Note 1). Mathematically speaking, $\pi_T = U_t \ \pi_s$ that is unitary operator U_t is applied on π_s and after time t; it would reach at π_T without hampering the system preserved in π_s.

Model 1. To evaluate targeted profit time when only cost- dependent parameter α is considered:

Consider an economy with a starting profit function π_s, revenue function R, and cost function C with cost- dependent parameter α acting on cost function only. Here there acts no parameter in revenue function R because we have taken a system where revenue function is unaltered. We can express the starting profit functions in the following way:

$$\pi_s = R - C\, e^{\alpha}. \tag{1}$$

Here we have considered the targeted profit π_T as a combination of revenue function which is same as the starting profit and cost function C. Here the cost- dependent parameter α acts negatively on cost function. Here a new firm faces a severe challenge with the old system and unable to increase the revenue remarkably. Consider the new revenue function R_1 and $R_1 = R + dR$. As dR is a negligible quantity, then $R_1 \approx R$. At this position firm manager has nothing to do only to reduce the cost function but as it is known that cost function and revenue function are correlated, so he will try to reduce the factors depending on cost dependent parameter α. For that reason, we have taken α as a negative one which acts on cost function. Then we write the targeted profit as,

$$\pi_T = R - C\, e^{-\alpha}. \tag{2}$$

Equations 1 and 2 give,

$$\begin{aligned} R &= \frac{\pi_T\, e^{\alpha} - \pi_I\, e^{-\alpha}}{e^{\alpha} - e^{-\alpha}} \\ &= \frac{\pi_T\, e^{\alpha} - \pi_I\, e^{-\alpha}}{2 \sin h\, \alpha} \end{aligned} \tag{3}$$

and

$$C = \frac{(\pi_T - \pi_S)}{2 \sin \mathrm{h}\, \alpha}. \tag{4}$$

[We get the results by using the properties, $\sinh \alpha = \frac{e^{\alpha} - e^{-\alpha}}{2}$.]

Equations (1) and (2) may be expressed in a matrix form as

$$\pi_s = \begin{pmatrix} R & C \\ -e^{\alpha} & 1 \end{pmatrix} \tag{5}$$

and

$$\pi_T = \begin{pmatrix} R & C \\ -e^{-\alpha} & 1 \end{pmatrix}. \tag{6}$$

So also equations (3) and (4) may be written as

$$C = \frac{1}{2 \sin h\, \alpha} \begin{pmatrix} \pi_T & -\pi_s \\ 1 & 1 \end{pmatrix} \tag{7}$$

and

$$R = \frac{1}{2 \sin h\, \alpha} \begin{pmatrix} \pi_T & \pi_s \\ -e^{-\alpha} & e^{\alpha} \end{pmatrix}. \tag{8}$$

Now set $U_t = \begin{pmatrix} \cos \omega t & \sin \omega t \\ -\sin \omega t & \cos \omega t \end{pmatrix}$, where ω is profit velocity.

Then, $U_t \ \pi_s = \begin{pmatrix} \cos \omega t & \sin \omega t \\ -\sin \omega t & \cos \omega t \end{pmatrix} \begin{pmatrix} R & C \\ -e^{\alpha} & 1 \end{pmatrix}$

$$= \begin{pmatrix} R \cos \omega t - e^{\alpha} \sin \omega t & C \cos \omega t + \sin \omega t \\ -R \sin \omega t - e^{\alpha} \cos \omega t & -C \sin \omega t + \cos \omega t \end{pmatrix}. \tag{9}$$

As we know $U_t \ \pi_s = \pi_T$ from the role of unitary operator of postulate 2.

That is, equating the equations (6) and (9) we get,

$$\begin{pmatrix} R \cos \omega t - e^{\alpha} \sin \omega t & C \cos \omega t + \sin \omega t \\ -R \sin \omega t - e^{\alpha} \cos \omega t & -C \sin \omega t + \cos \omega t \end{pmatrix} = \begin{pmatrix} R & C \\ -e^{-\alpha} & 1 \end{pmatrix}. \tag{10}$$

Equating the both sides of equation (10) we get the only valid equation

$C \cos \omega t + \sin \omega t = C$, where other equations give negative time or improper equation of time. Now from $C \cos \omega t + \sin \omega t = C$, we get

$$t = \frac{2}{\omega}\tan^{-1}\frac{1}{C}. \tag{11}$$

Model 2: To find out time for targeted profit function when revenue increasing parameter is considered:

Now if we express starting and targeted profit functions in the following way,

$$\pi_s = R\,e^{-\beta} - C \tag{12}$$

and

$$\pi_T = R\,e^{\beta} - C, \text{ where } \beta \text{ is revenue increasing factor.} \tag{13}$$

This gives the values of revenue and cost function as

$$R = \frac{\pi_T - \pi_s}{e^{\beta} - e^{-\beta}} = \frac{\pi_T - \pi_s}{2\ \sin h\ \beta} \tag{14}$$

and

$$C = \frac{\pi_T\, e^{-\beta} - \pi_s\, e^{\beta}}{e^{\beta} - e^{-\beta}} = \frac{\pi_T\, e^{-\beta} - \pi_s\, e^{\beta}}{2\ \sin h\ \beta}. \tag{15}$$

Now π_s and π_T can be written in the matrix form as,

$$\pi_s = \begin{pmatrix} R & -C \\ 1 & e^{-\beta} \end{pmatrix} \tag{16}$$

and

$$\pi_T = \begin{pmatrix} R & -C \\ 1 & e^{\beta} \end{pmatrix} \tag{17}$$

Again from postulate 2 we get,

$$U_t\ \pi_s = \begin{pmatrix} \cos\omega t & \sin\omega t \\ -\sin\omega t & \cos\omega t \end{pmatrix}\begin{pmatrix} R & -C \\ 1 & e^{-\beta} \end{pmatrix}$$
$$= \begin{pmatrix} R\cos\omega t + \sin\omega t & -C\cos\omega t + e^{-\beta}\sin\omega t \\ -R\sin\omega t + \cos\omega t & -C\sin\omega t + e^{-\beta}\cos\omega t \end{pmatrix} \tag{18}$$

Now from the relation $U_t\ \pi_s = \pi_T$ we obtain,

$$\begin{pmatrix} R\cos\omega t + \sin\omega t & -C\cos\omega t + e^{-\beta}\sin\omega t \\ -R\sin\omega t + \cos\omega t & -C\sin\omega t + e^{-\beta}\cos\omega t \end{pmatrix} = \begin{pmatrix} R & -C \\ 1 & e^{\beta} \end{pmatrix}.$$

Equating corresponding elements of both matrices we get the legitimate equation $R\cos\omega t + \sin\omega t$ = R leads to

$$t = \frac{2}{\omega}\tan^{-1}\frac{1}{R}. \tag{19}$$

Model 3: To find out time for targeted profit function when both revenue increasing parameter and cost increasing parameter are considered:

Again if we set the starting and targeted profit in the following way,

$$\pi_s = R\,e^{-\beta} - C\,e^{\alpha} \tag{20}$$

and

$$\pi_T = R\,e^{\beta} - C\,e^{-\alpha}. \tag{21}$$

Here we get revenue and cost function as,

$$R = \frac{\pi_T e^{\alpha} - \pi_s e^{-\alpha}}{2\sin h\,(\alpha+\beta)} \tag{22}$$

and

$$C = \frac{\pi_T e^{-\beta} - \pi_s e^{\beta}}{2\sin h\,(\alpha+\beta)}. \tag{23}$$

Now we express π_s and π_T in the matrix form as,

$$\pi_s = \begin{pmatrix} R & C \\ -e^{\alpha} & e^{-\beta} \end{pmatrix} \tag{24}$$

and

$$\pi_T = \begin{pmatrix} R & C \\ -e^{-\alpha} & e^{\beta} \end{pmatrix}. \tag{25}$$

By using postulate 2 we get,

$$U_t\,\pi_s = \begin{pmatrix} \cos\omega t & \sin\omega t \\ -\sin\omega t & \cos\omega t \end{pmatrix}\begin{pmatrix} R & C \\ -e^{\alpha} & e^{-\beta} \end{pmatrix}$$

$$= \begin{pmatrix} R\cos\omega t - e^{\alpha}\sin\omega t & C\cos\omega t + e^{-\beta}\sin\omega t \\ -R\sin\omega t - e^{\alpha}\cos\omega t & -C\sin\omega t + e^{-\beta}\cos\omega t \end{pmatrix} \quad \ldots (26)$$

The relation $U_t\ \pi_s = \pi_T$ gives,

$$\begin{pmatrix} R\cos\omega t - e^{\alpha}\sin\omega t & C\cos\omega t + e^{-\beta}\sin\omega t \\ -R\sin\omega t - e^{\alpha}\cos\omega t & -C\sin\omega t + e^{-\beta}\cos\omega t \end{pmatrix} = \begin{pmatrix} R & C \\ -e^{-\alpha} & e^{\beta} \end{pmatrix}$$

leads to $t = \frac{2}{\omega}\tan^{-1}\left(\frac{e^{-\beta}}{C}\right)$.

Conclusion

It is observed that with the application of different unitary operators a starting profit function of a firm may achieve its final destination in arbitrarily short time by using Unitary Operator. Now a million dollar question arises whether such an operation is implemented in practice? If the answer is affirmative then it has an immense implication in the profit maximization policy of a firm and in other related fields.

If managers of a firm study the characteristics of the profit velocities and observes the conditions of the starting profit function and also study the trajectory path of the destination point then it is possible to find out the least time to reach the target through a well-developed computer.

Note

1. A matrix U is said to be unitary if $U^T U = I$, where I is a unit matrix. Similarly an operator U is unitary if $(U^T)^* U = I$, where U^* is adjoint of U and I is the identity operator. It is easily checked that an operator is unitary if and only if each of its matrix representations is unitary. The property of a unitary operator is that if a vector is acted on by a unitary operator, the length of the vector remains unchanged. The corresponding physical meaning is that no new system would be created or the system in π_s would not collapse suddenly under unitary evolution. For that reason we have chosen unitary operator.

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